

Phase Synchronization in a Plasma Discharge Driven by a Chaotic Signal

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Abstract. Two different coupled experimental systems (the Chua circuit and a plasma discharge) are demonstrated to be capable of phase synchronizing to each other. Real-time observation of power spectra in association with oscilloscope plotting of the signals of both systems allows the visualization of the transition from unsynchronization to synchronization.

INTRODUCTION

Synchronization is not a new phenomenon. The repeatedly occurrence of simultaneous events has long been observed in nature and also used in many types of devices [1]. It goes back to Huygens [2] who in 1673 reported the synchronous motion of two pendulum clocks hanging on the same wood beam. More recently, in the case of chaotic dynamical systems, synchronization has become an area of great interest. Initially expected not to be able to achieve synchronization [3], chaotic systems can indeed synchronize depending on the type of coupling, parameter values, and other factors. In particular, chaotic systems may be coupled in ways that allow the synchronization of their phases only, keeping amplitudes completely uncorrelated [4]. Phase synchronized chaos has received much attention lately with major theoretical and experimental work [5]. This includes the instance of a chaotic system forced with a periodic oscillator [6] demonstrated experimentally with a plasma discharge tube paced with a sinusoidal function [7], and with a music signal [8]. In all these cases there is a need for a clear understanding of what is meant by phase of the oscillator [9]. In the case of a circular oscillator, phase can be defined as an angular variable ϕ increasing monotonically in time on the limit cycle: $d\phi/dt = \omega$, where $2\pi/\omega = T$ is the period of the oscillation. This definition is suitable for chaotic oscillators, such as the Rössler system [10], by introducing a phase angle variable. We take the projection of the attractor on the (x, y) plane and define the phase $\phi(t) = \arctan[y(t)/x(t)]$. For experiments where simultaneous recording of multiple measurements may restrict real time observation of attractors, the Hilbert transform can be useful for studying phase synchronization. It allows the computation of an analytical signal corresponding to each rotation, yielding the phase for that rotation. Using the analytical signal concept introduced by Gabor [11], the instantaneous phase of a signal $s(t)$ is $\phi(t) = \arctan[\tilde{s}(t)/s(t)]$, where $s(t) + i\tilde{s}(t) = A(t)e^{i\phi(t)}$, and

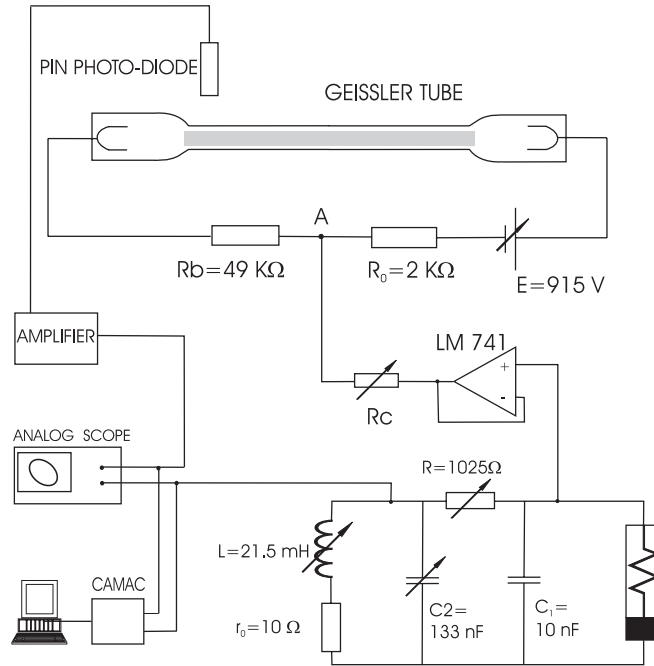


FIGURE 1. Chua-plasma experimental setup.

$\tilde{s}(t) = \pi^{-1} \text{P.V.} \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau} d\tau$. Here $A(t)$ is the instantaneous amplitude and P.V. stands for the principal value of the integral.

In this paper we propose to discuss the results of experimental measurements involving a plasma discharge tube coupled to a Chua electronic circuit. The coupling is unidirectional and works with a voltage from the Chua circuit fed into the plasma. We verify that, depending on the strength of the coupling, the two systems can phase synchronize for long time intervals. A special technique allows real time verification of phase synchronization, and proves to be very useful for testing of parameter values associated with different regimes.

EXPERIMENTAL SETUP

The setup of our experimental arrangement is shown in Fig. 1. The unidirectional coupling between plasma and Chua is achieved through an op-amp LM741, and a current limiting variable resistor R_c . This resistor is used to control the strength of the coupling. Increasing the resistance reduces the current with consequent weak coupling, and vice-versa. The Chua circuit [12] has $R = 1025 \Omega$, $C_1 = 10 \text{ nF}$, $C_2 = 133 \text{ nF}$, $L = 21.5 \text{ mH}$, and $r_0 = 10 \Omega$. The voltage across the capacitor C_1 is the driving signal injected into the plasma. The plasma is produced in a commercial Geissler tube filled with helium gas. The discharge is powered by a high voltage source set at $E \simeq 900 \text{ V}$, and sustained by the ballast resistor $R_b = 49 \text{ k}\Omega$. The light intensity oscillations of the plasma column is acquired using a PIN photo-diode.

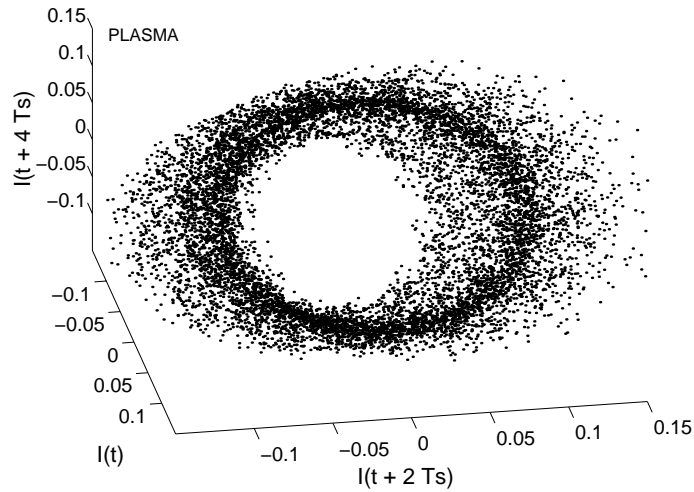


FIGURE 2. (a) Plasma attractor.

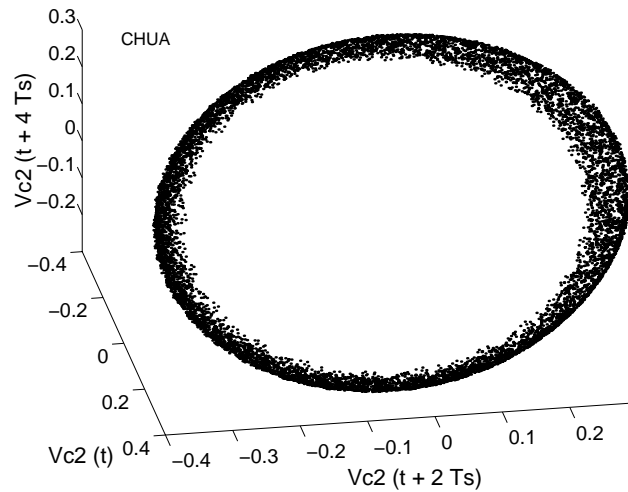


FIGURE 3. (a) Chua attractor.

TUNING THE OSCILLATORS

The signal acquired from the oscillations of the plasma light intensity can be plotted real time on an oscilloscope. If this signal is plotted versus its analog derivative, an attractor can be viewed and used for tuning the system. Alternatively, time delay embedding can also be used to produce the corresponding attractor, as pictured in Fig. 2. The same can be done for the voltage across C_2 of the Chua circuit, with the resulting attractor shown in Fig. 3. The plasma and the Chua signals originate from different systems but have similar broad power spectra with a peak indicating a dominant frequency [13]. Figure 4 depicts both spectra displaying a mismatch of about 50 Hz between the peaks when the two oscillators are uncoupled. A real time visualization of these two signals can be obtained by plotting one versus the other on the oscilloscope. An ellipse-like

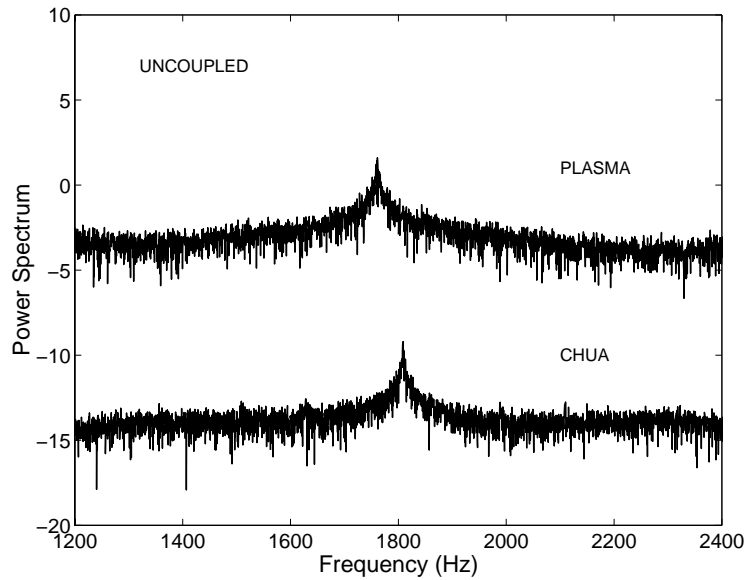


FIGURE 4. Spectra of uncoupled plasma and Chua.

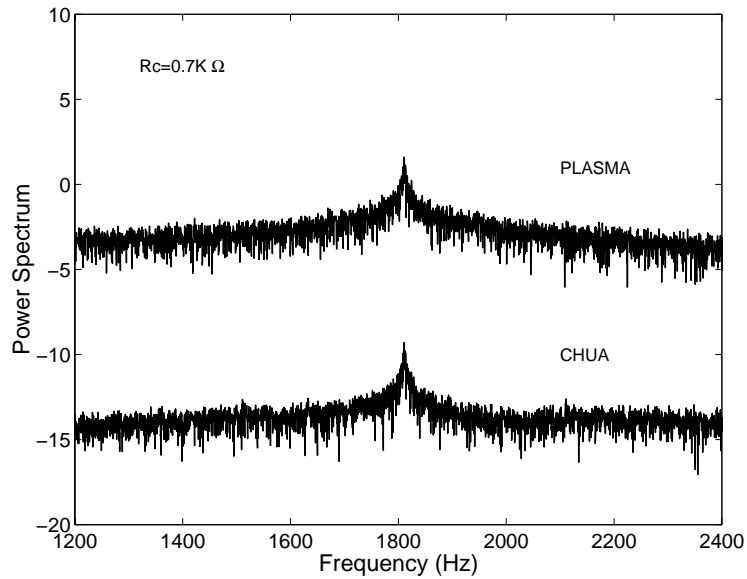


FIGURE 5. Spectra of coupled plasma and Chua.

figure is obtained oscillating back-and-forth between the angles of 45° and 135° . This back-and-forth motion is a clear indication that the plasma and the Chua are not phase synchronized. However, by coupling the two systems and tuning the circuit to reduce the mismatch between the dominant frequencies on the spectra, we observe that the ellipse on the oscilloscope becomes more stable and ceases oscillating when the frequency peaks get very close to each other. Figure 5 shows the spectra of both signals with Chua and plasma coupled and very small mismatch between the dominant frequencies of the

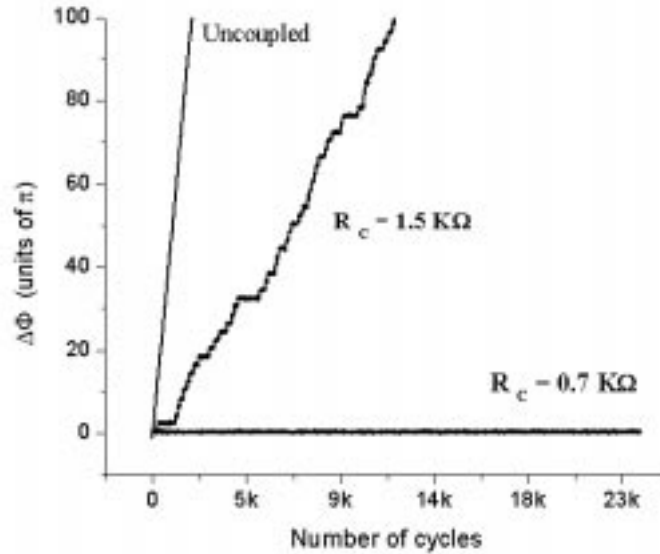


FIGURE 6. Phase synchronization ($R_c = 0.7\text{k}\Omega$), phase slips of 2π ($R_c = 1.5\text{ k}\Omega$), and phase unsynchronization (uncoupled).

two oscillators. Notice how the dominant frequency of the plasma moves toward the dominant frequency of the Chua when the coupling is turned on. In this case the ellipse-like plot of one signal versus the other is stable and does not oscillate, typical indication that the two oscillators are phase synchronized.

The coupling resistor R_c plays an important roll in the process when the two oscillators go from phase synchronized to phase unsynchronized chaos. This can be observed in Fig. 6 where the phase difference $\Delta\Phi$ between plasma and Chua is plotted versus the number of cycles. If the oscillators are not coupled, notice that $\Delta\Phi$ grows very fast. However, if the coupling is on with $R_c = 1.5\text{ k}\Omega$, then $\Delta\Phi$ still grows, but shows plateaus where the two oscillators have their phases synchronized during short time intervals. Finally, when $R_c = 0.7\text{ k}\Omega$, the coupling is stronger and in this case the phases of the two oscillators remain synchronized for very long time intervals.

CONCLUSION

In conclusion, we have developed a technique for experimental real-time observation of synchronized states between a plasma discharge and a Chua circuit. The power spectra of the two systems associated with the oscilloscope observation of the plasma versus Chua signals provides a concrete and reliable manner for checking the state of the two oscillators. Depending on the strength of the coupling, we observe slips of 2π between the phases of the two oscillators.

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